

# Uncertainty Analysis for a Jet Flap Airfoil

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**ABSTRACT:** *An analysis of variance (ANOVA) study was performed to quantify the potential uncertainties of lift and pitching moment coefficient calculations from a computational fluid dynamics code, relative to an experiment, for a jet flap airfoil configuration. Uncertainties due to a number of factors including grid density, angle of attack and jet flap blowing coefficient were examined. The ANOVA software produced a numerical model of the input coefficient data, as functions of the selected factors, to a user-specified order (linear, 2-factor interference, quadratic, or cubic). Residuals between the model and actual data were also produced at each of the input conditions, and uncertainty confidence intervals (in the form of Least Significant Differences or LSD) for experimental, computational, and combined experimental / computational data sets were computed. The LSD bars indicate the smallest resolvable differences in the functional values (lift or pitching moment coefficient) attributable solely to changes in independent variable, given just the input data points from selected data sets. The software also provided a collection of diagnostics which evaluate the suitability of the input data set for use within the ANOVA process, and which examine the behavior of the resultant data, possibly suggesting transformations which should be applied to the data to reduce the LSD. The results illustrate some of the key features of, and results from, the uncertainty analysis studies, including the use of both numerical (continuous) and categorical (discrete) factors, the effects of the number and range of the input data points, and the effects of the number of factors considered simultaneously.*

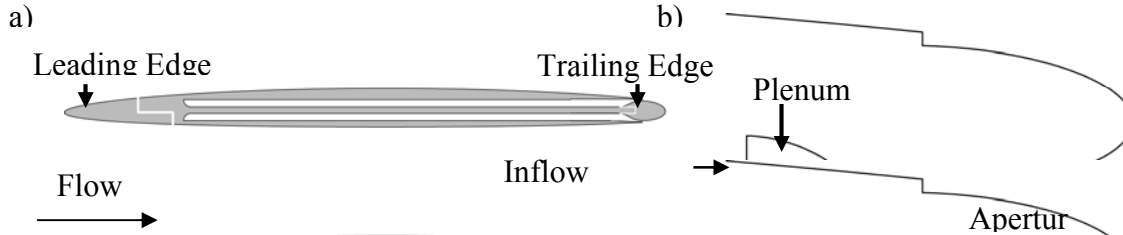


Figure 2.1 a) Cross Section, b) Trailing Edge Schematic and Plenum.

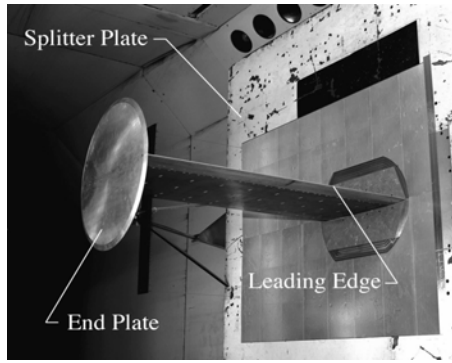
## 1. Introduction

The application of a Computational Fluid Dynamics (CFD) tool to a jet flap control effector on an elliptical airfoil-section wing has been investigated [1]. The Tetrahedral Unstructured Software System (TetrUSS) [2-3] was used in this investigation. The system was developed at NASA Langley Research Center and includes the Reynolds-averaged Navier-Stokes flow solver code USM3D [4]. The CFD-based jet flap simulations were compared to experimental results from a wind-tunnel test that was conducted in NASA Langley Research Center's Transonic Dynamics Tunnel [5]. An analysis of Variance (ANOVA) study was then conducted using the Design-Expert software (version 6.0.10) from Stat-Ease, Inc. [6-10] in order to quantify the uncertainty of the computational results relative to the experimental

results. Uncertainties associated with several parameters were analyzed. Other ANOVA studies were conducted and reported for other applications in Reference [11].

## 2. Experimental Description

As shown in Figure 2.1, the model consisted of a modified six percent thick elliptical two-dimensional airfoil with 0.75% circular-arc camber and zero leading and trailing edge sweep. The jet flap was located at 95% chord and was created by exhausting a stream of high-pressure air from a lower surface slot that was close to the trailing edge (Fig. 2.1a). The jet sheet exited at 90° degrees to the chord axis (Fig. 2.1b). The model included a circular end plate of 30 in. diameter to promote two-dimensional flow across the wing and a splitter plate that offset the model 40 in. from the tunnel wall (Fig. 2.2).



**Figure 1.2. Experimental Model.**

It was found the CFD simulation had to model the three-dimensional geometry of the experiment in order to obtain good agreement. Tests were performed at two Mach numbers at several different jet momentum coefficients ( $C_{\mu}$ ). In order to be consistent with the experiment method, the CFD lift and pitching moment values were found by integrating the pressures over the wing.

The wing was instrumented with a total of 157 static and total pressures taps of which 93 (47 upper & 46 lower) external static surface pressure taps were located at  $y/b=0.5$  on the upper and lower airfoil surface. The sectional lift and moment coefficients were calculated by integrating the pressure measurements at this location. The model did not include a force balance. Test were performed at Mach=0.3 at a Reynolds of 400,000/ft and Mach=0.8 at a Reynolds of 900,000/ft. The momentum coefficients,  $C_{\mu}$ , for the jet flap data ranged from 0.006 to 0.067.

### 3. Computational Description

TetrUSS is a software system consisting of four modules. It is a loosely integrated unstructured-grid CFD system that provides ready access to rapid higher-order analysis and design capability for the applied aerodynamicist. TetrUSS uses Gridtool [12] for geometry setup, VGRID [3] for grid generation, USM3D [4] for the flow solver, and VGLOT [3] for post-processing. The code uses a cell centered, upwind biased, finite volume and implicit/explicit algorithm to solve the compressible Euler and Navier-Stokes equations on an unstructured tetrahedral mesh [4,13-14]. All the results obtained for this CFD simulation were performed using the Spalart-Allmaras [15] turbulence model.

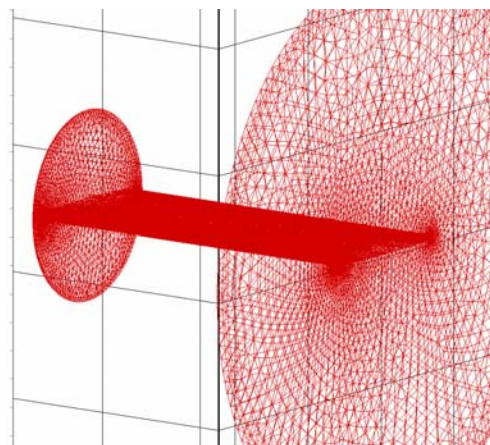
All the computations were performed using the NASA Ames Columbia supercomputer system. Typically, from 7 to 28 wall clock hours were needed for each run, depending on size of the grid, with 96 processors working as a parallel system. The required CPU usage ranged from 660 to 2700 hours of processor time per case.

Convergence criteria were based on lift, pitching moment, and drag histories as well as the residual convergence.

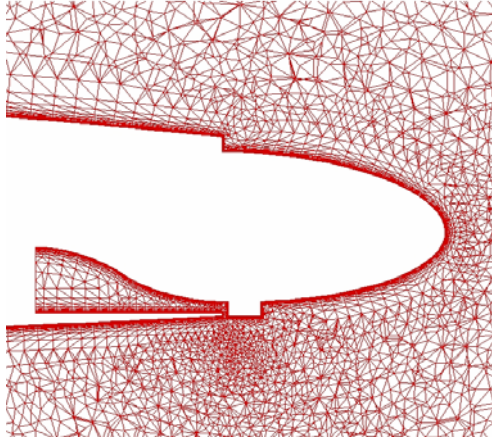
Generating the grid was difficult because of the small-scale features imbedded in the geometry, as shown in Figure 3.1. The aperture and surrounding region (shown in Figures 3.2 and 3.3) were the biggest problem areas for the grid generation. Five different grid densities were generated and used to perform a sensitivity study in order to assess if asymptotic behavior was observed of the sectional lift coefficient,  $C_l$ , as a function of a grid density metric. Such asymptotic behavior would indicate that grid independence of the solution was achieved. It was determined in Ref. 1 that the second-most dense grid of the five provided sufficient fidelity and accuracy.

The jet flap behaves like a mechanical flap, specially, a gurney flap. It can be seen in Figures 3.4 and 3.5 that the jet flap creates an area of lower Mach numbers upstream of it and deflects the streamlines downward just as a small physical flap would. The obvious advantage of the jet flap is that a mechanical system is not needed. However, one important disadvantage of the jet flap is that it has decreasing control authority as dynamic pressure increases whereas a mechanical flap would have increasing authority with higher dynamic pressure.

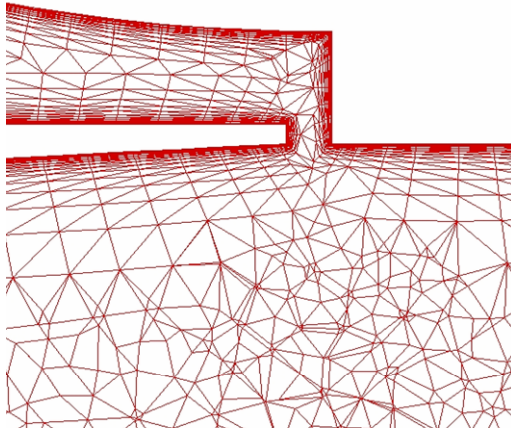
The blowing influences the velocities over the entire wing. In transonic flow at Mach=0.8, the blowing induces a change in the shock location on the upper surface of the airfoil, making for larger, nonlinear changes in the measured or computed lift and pitching moment coefficients. While there is no shock at low subsonic Mach numbers, a similar global effect on the flow field occurs at the Mach=0.3 condition.



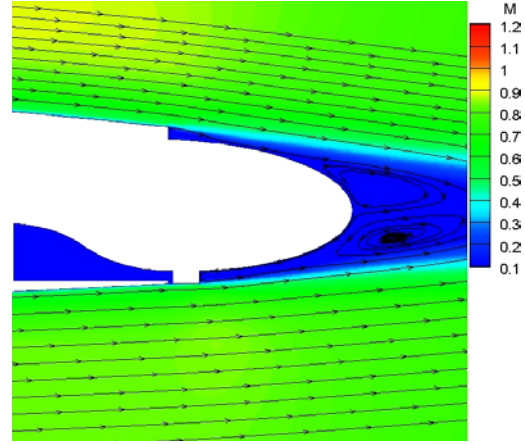
**Figure 2.1 Grid Model.**



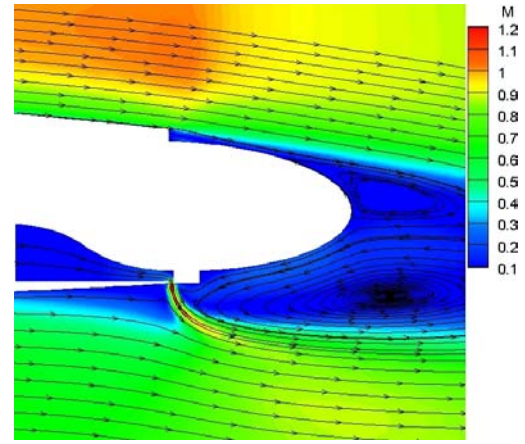
**Figure 3.2 Cross Section of the 3D Model and Grid at the Trailing Edge.**



**Figure 3.3 Aperture Cross Section Close-Up**



**Figure 3.4 Trailing Edge Flow Visualization at Mach=0.8, Alpha=6.0°,  $C_{\mu}$ =0.0000.**



**Figure 3.5 Trailing Edge Flow Visualization At Mach=0.8, Alpha=3.0°,  $C_{\mu}$ =0.0115.**

#### 4. Uncertainty Analysis

An Analysis of Variance (ANOVA) study was conducted using the Design-Expert software (version 6.0.10) from Stat-Ease, Inc. [6-11] in order to quantify the uncertainty of the computational results due to grid density, angle of attack, and  $C_{\mu}$ . Uncertainty analysis was also performed for the experimental data for potential uncertainties due to  $C_{\mu}$  and to quantify the relative uncertainties between the computational and experimental results.

To perform the ANOVA, factors such as the Mach number, blowing coefficient, and a grid density metric, which potentially affect the computed or measured lift and pitching moment coefficient, were identified and input to the software. Ranges of interest for the factors were input to the software along with the discrete computed or measured values of the factors. Also input to the software were the computed or measured lift and

pitching moment coefficient responses associated with the input values of the factors. In a matter of seconds, using a desktop workstation, a numerical model of user-specified order (linear, 2-factor interference, quadratic, or cubic) was calculated to fit the input data, residuals between the model and actual data were produced at each of the input conditions, and uncertainties (in the form of Least Significant Differences or LSD) for the experimental, computational, and combined data sets were computed. The LSD computed by the software indicate the smallest resolvable differences in the functional values (lift and pitching moment coefficient) given just the input data points from selected data sets. The software also provides a collection of diagnostics which evaluate the suitability of the input data set for use within the ANOVA process and which examine the behavior of the resultant data, suggesting transformations which should be applied to the data to reduce the LSD.

Figures 4.1 to 4.9 illustrate some of the key features and results from the uncertainty analysis studies. Figure 4.1 illustrates the uncertainty analysis for the grid sensitivity study, described previously, at Mach = 0.3. The curve in the plot represents the approximate quadratic numerical model, which the Design-Expert software fitted to the input data. Design points are shown where input data were provided. The lift coefficient is shown as a function of the grid density metric with 95% confidence intervals in the form of least significant difference (LSD) error bars added; the blue square simply indicates where on the curve the LSD is applicable. The LSD bars indicate the smallest resolvable differences in the functional values (lift coefficient) attributable to changes in the grid density metric, given just the selected input data points. In this case, the LSD is about 0.0050, which is divided by two and added or subtracted from the calculated numerical model value to obtain high and low 95% confidence bounds. This means the potential lift error due to grid density is about  $\pm 0.0025$ .

On the finest grid, the actual lift coefficient at this grid density (0.7750) is expected to be between 0.7724 and 0.7775 with 95% confidence; there is a 5% chance that the actual lift on this grid could be outside of these bounds. The actual lift coefficient, in this case, is well within the 95% confidence bounds. The actual lift coefficient value on the selected (second-most dense) grid (0.7741, second from left) also lies well within the 95% confidence interval for that grid, where the approximate model value is 0.7744, and the range is between 0.7719 and 0.7769 with 95% confidence. However, the parabolic shape of the approximate model does not represent the expected asymptotic behavior of the data, which causes an ambiguity in the model for this region, and also causes the confidence interval to be broader than might be expected and broader than would be desired. As shown in Figure 4.2, the uncertainty can be reduced on the selected grid, simply by ignoring the finest grid results; this removes ambiguity about the behavior of the approximate lift coefficient model as the grid density is increased (moving toward left on the plot), and allows for a prediction of the actual lift coefficient on the selected grid to be within  $\pm 0.0004$  of the approximate model value 0.7741, or that the actual lift coefficient is expected to be between 0.7737 and 0.7745, with 95% confidence. This is a reduction of the prediction uncertainty by more than a factor of ten, by simply removing some ambiguity in the way the data was presented to, and modeled by, the software.

Figure 4.3 presents similar results for the grid sensitivity study at Mach = 0.8. In this case, a linear numerical model was fit to the input data, again excluding the data from the finest grid. The 95% confidence LSD in this case is about 0.0070.

Figure 4.4 illustrates a different way to perform the grid sensitivity uncertainty analysis. In this example, both the Mach = 0.3 and Mach = 0.8 have been considered together, with data from the finest grid again excluded. The lift coefficient was described to the Design-Expert software as a function of the numerical (continuous) grid density factor and a categorical factor of Mach number, which could only take on the discrete values of 0.3 or 0.8. Since the lift behavior with grid density for both grid studies has the same functional form, the software can better resolve the potential error due to grid density. The maximum LSD of the combined data set is now about 0.0016.

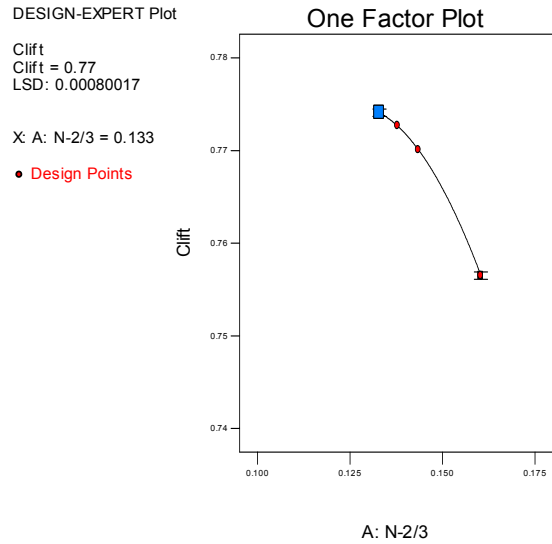
Figures 4.5 and 4.6 illustrate the combined uncertainty analysis for angle of attack and blowing coefficient, based upon Mach = 0.3 cases at  $\alpha = 3^\circ$  and  $6^\circ$  angle of attack, and Mach = 0.8 cases for  $\alpha = 3.0^\circ$  and  $3.1^\circ$  angle of attack, respectively. The LSD due to angle of attack is about 0.0096 and 0.0301, respectively, for the two groups of different Mach data. In the Mach = 0.3 case, data are provided at two values of blowing coefficient. For the Mach = 0.8 case, data are provided at three values of blowing coefficient, but these data points are not co-linear and thus introduce ambiguity into the linear model which broadens the LSD more than might be expected. But even only data at two alpha and two blowing coefficient are provided, as is the case for the Mach = 0.3 data, the two-factor linear model (three terms) cannot perfectly represent the data, which is the source of a non-zero LSD for this data set. The data points form a “warped” plane; the corners can all be connected by straight lines, but the slope of the lines on each end of the sheet are different.

Figure 4.7 illustrates the uncertainty analyses for two similar, but different, input data sets. Both computational and experimental Mach = 0.3 data with blowing coefficient up to about 0.07 are analyzed as a function of the blowing coefficient,  $C_\mu$ . The difference between computational and experimental data is treated again as a categorical factor, which can only take on two distinct values (i.e., Type = Computational or Experimental), whereas the blowing coefficient is treated as a numerical (continuous) factor. The LSD is about 0.1106. Similar analysis for the Mach=0.8 data (not shown), with blowing coefficient up to 0.0240 yielded an LSD of about 0.0747.

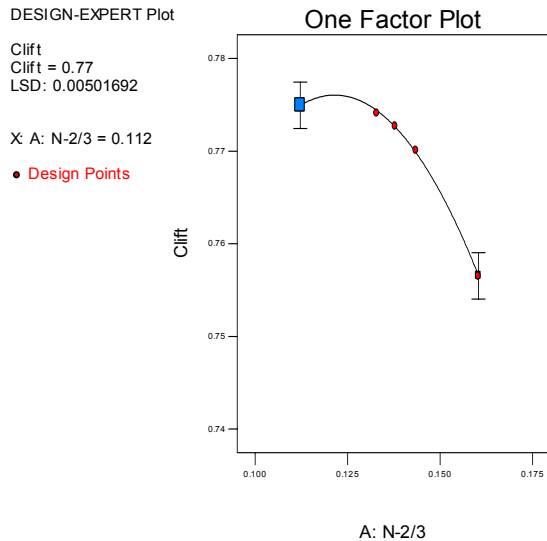
The same analysis as in Figure 4.7 is repeated in Figure 4.8 for Mach = 0.3, but with only incremental jet effects (Jet On – Jet Off) now considered. That is, the lift augmentation behavior reported in Ref. 1, Figure 18, is now analyzed as a function of the blowing coefficient. In this case, the LSD is about 0.0838, but the software has recommended that a power transformation be applied to the data. Figure 4.8 shows the untransformed data,



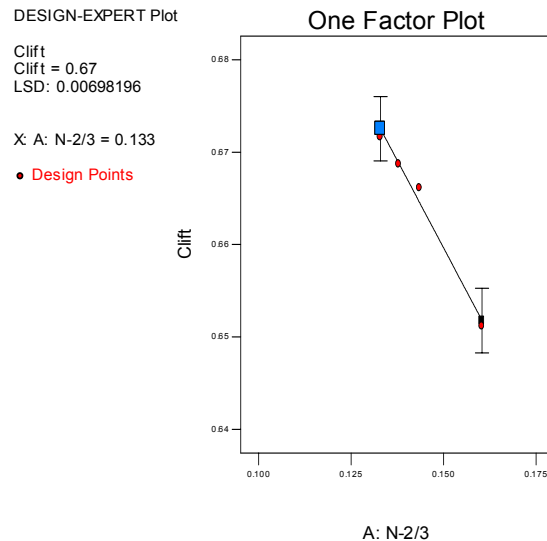
whereas Figure 4.9 shows the transformed Mach = 0.3 data; a constant of 0.00684186 was added to each input lift coefficient and the sum was raised to the 1.63 power, and refit in this transformed space. The LSD is now reduced to 0.0113 in the transformed space, which can be shown to reduce the LSD to about 0.0067 in the untransformed space. Therefore, a significant reduction in lift coefficient uncertainty is found using the incremental jet effects, combined with a power transformation that was recommended by the software. Similar jet increment lift augmentation, using a transformation with constant of 0.00399084 and a power of 1.68 for the Mach = 0.8 data (not shown), resulted in an actual LSD of about 0.0135, which again implies there is more uncertainty sensitivity due to the nonlinear effect of shock movement with changes in blowing coefficient.



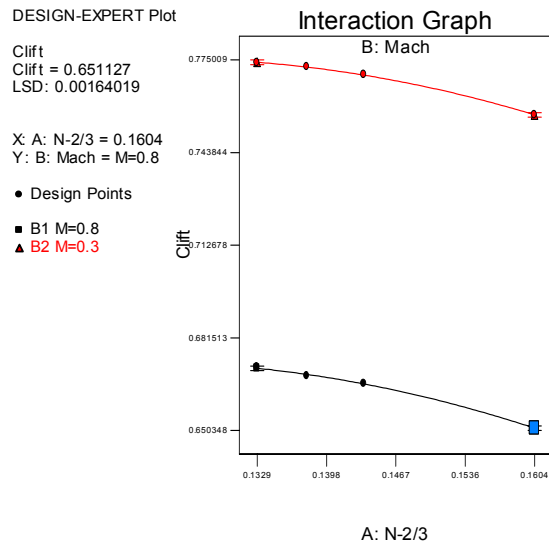
**Figure 4.2 Uncertainty Analysis, Grid Sensitivity**  
Study at Mach = 0.3, Alpha = 6.0°,  
finest grid data excluded.



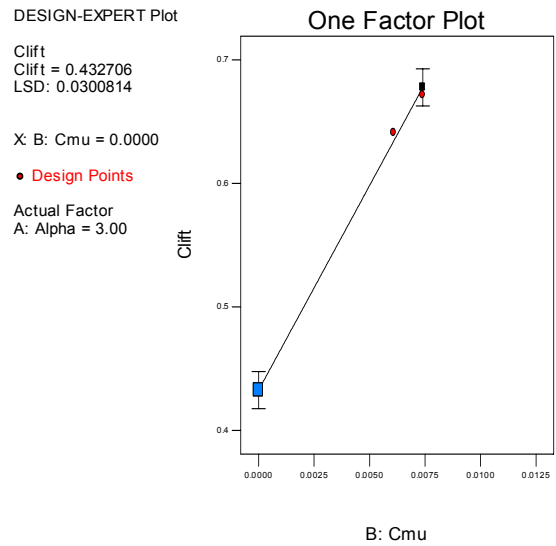
**Figure 4.1 Uncertainty Analysis, Grid Sensitivity**  
Study at Mach = 0.3, Alpha = 6.0°.



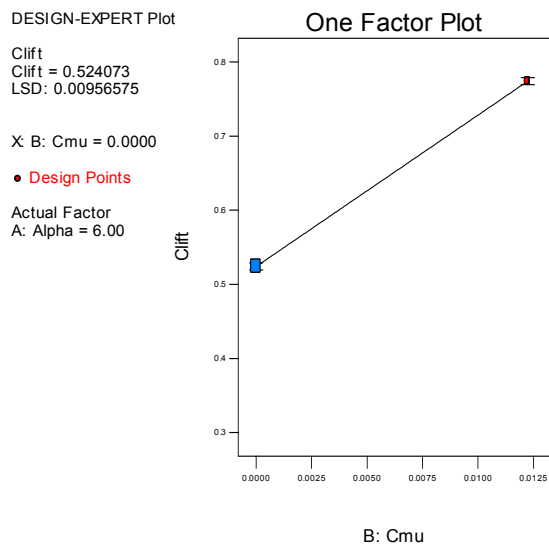
**Figure 4.3 Uncertainty Analysis, Grid Sensitivity**  
Study at Mach = 0.8, Alpha = 3.0°,  
finest grid data excluded.



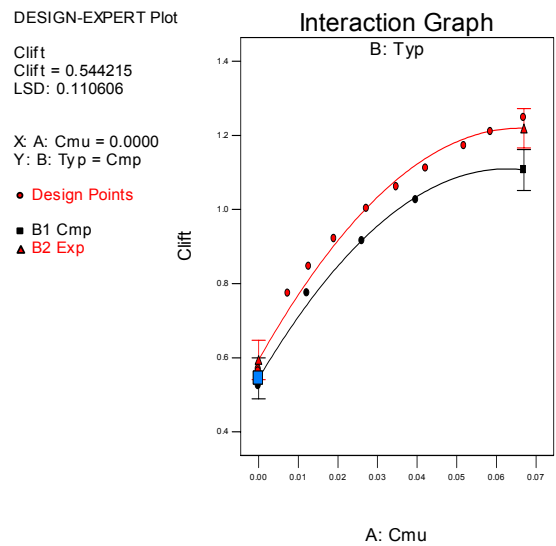
**Figure 4.4 Uncertainty Analysis, Grid Sensitivity Study at Mach = 0.3 and Mach = 0.8, finest grid data excluded.**



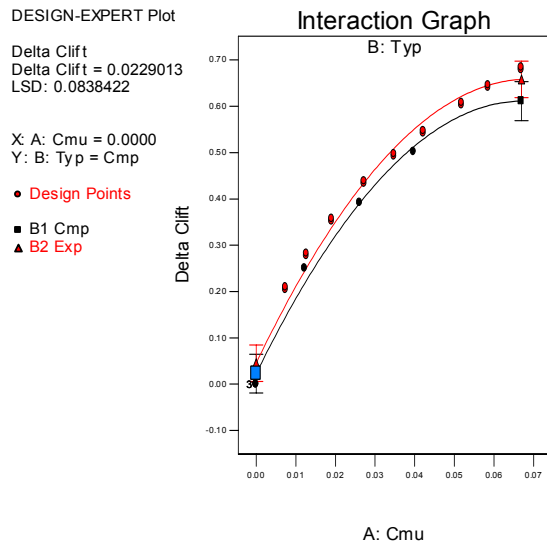
**Figure 4.6 Uncertainty Analysis, Angle of Attack Sensitivity Study at Mach = 0.8, Alpha = 3.0°.**



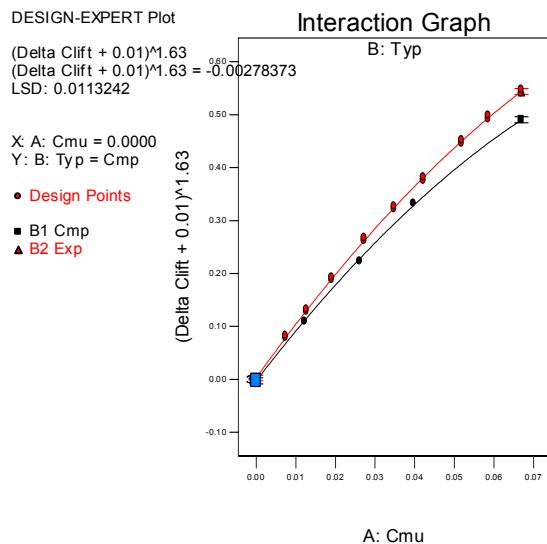
**Figure 4.5 Uncertainty Analysis, Angle of Attack Sensitivity Study at Mach = 0.3, Alpha = 6.0°.**



**Figure 4.7 Uncertainty Analysis, Blowing Coefficient Sensitivity Study at Mach = 0.3, Alpha = 6.0°.**



**Figure 4.8 Uncertainty Analysis, Blowing Coefficient Sensitivity Study at Mach = 0.3, Alpha = 6.0°, repeated experimental data included, no transformation.**



**Figure 4.9 Uncertainty Analysis, Blowing Coefficient Sensitivity Study at Mach = 0.3, Alpha=6.0°, repeated experimental data included, power transformation applied,  $k = 0.00684186$ ,  $\lambda = 1.63$ .**

## 5. Conclusions

An analysis of variance (ANOVA) study was performed to quantify the potential uncertainties of lift and pitching moment coefficient calculations from a computational fluid dynamics code, relative to an experiment for a jet flap airfoil configuration at Mach = 0.3 and 0.8.

Uncertainties due to a number of physical factors including grid density, angle of attack and jet flap blowing coefficient were examined. Experimental, computational, and combined sets of experimental/computational data were analyzed. The use of both numerical (continuous) and categorical (discrete) factors, the effects of the number and range of the input data points, and the effects of the number of factors considered simultaneously, were all examined. Uncertainty bounds or confidence intervals (in the form of Least Significant Differences or LSD) were presented which represent the smallest resolvable differences in the functional values attributable solely to changes in independent variable, given just the input data points from selected data sets. In general, the data at Mach = 0.8 exhibited greater levels of uncertainty than did the Mach = 0.3 data for similar uncertainty quantification studies. The uncertainty bounds were also shown to be highly dependent upon the specific characteristics of the data set(s) analyzed including the effects of purposely repeated and otherwise indistinguishable points. Diagnostics were used to evaluate the suitability of the input data sets for use within the ANOVA process, to examine the behavior of the resultant data, and to suggest transformations that should be applied to the data to reduce the LSD. The entire package of capabilities was found to be extremely useful and easy to apply in wide variety of analyses.

## 6. References

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**JOSUE CRUZ** has been a civil servant at NASA Langley Research Center for 2 years. He obtained his Bachelor's (2002) and Master's degree (2004) in Mechanical Engineering at the California State University, Los Angeles (CSULA). At CSULA he participated in a project for NASA and Air Force as a research assistant for three years. He is currently working for the Advanced Aerospace System Branch; present work involves the analysis of unconventional control effectors using CFD, and wind tunnel testing.